# FINAL MARKING GUIDE
## 1 June 2011

| Rossmoyne SHS Mathematics Department | MATHEMATICS SPECIALIST 3A | Semester 1 2011 EXAMINATION |
|---|---|---|

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NAME: **SOLUTIONS**

TEACHER (circle) : **Mr White  Ms Rigelsford
Ms Belonogoff  Ms Robinson  Mr Jones**

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# Section Two:
# Calculator-assumed

**NOTE:**
Rounding will only be penalised (max −1) for Q8 ONLY.

Incorrect UNITS will only be penalised (max −1) for Q10 ONLY

## Time allowed for this section
Reading time before commencing work:  10 minutes
Working time for this section:  100 minutes

## Materials required/recommended for this section
*To be provided by the supervisor*
This Question/Answer Booklet
Formula Sheet (retained from Section One)

*To be provided by the candidate*
Standard items:  pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items:  drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

## Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available |
|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 40 |
| Section Two: Calculator-assumed | 11 | 11 | 100 | 80 |
| | | | | 120 |

## Instructions to candidates

1.  The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.

2.  Answer the questions according to the following instructions.

    Section Two: Write answers in this Question/Answer Booklet. All questions should be answered.

    **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

    It is recommended that you **do not use pencil** except in diagrams.

3.  You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4.  Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

| QUESTION | MARKS AVAILABLE | STUDENT MARK |
|----------|-----------------|--------------|
| 8 | 7 | |
| 9 | 4 | |
| 10 | 10 | |
| 11 | 12 | |
| 12 | 6 | |
| 13 | 10 | |
| 14 | 6 | |
| 15 | 6 | |
| 16 | 7 | |
| 17 | 6 | |
| 18 | 6 | |
| TOTAL | 80 | |

**Question 8**
[ 7 marks]

Robert is asleep in his chair and unbeknown to him Tom, Dylan and Michael have each tied ropes around him and begin to pull with the forces and directions shown.

In which direction and with what force does the sleeping Robert move? Give the direction as a bearing. $(1dp)$
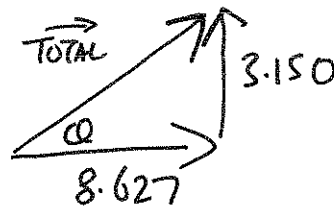
North

Dylan

Tom

10N   40°   20N

10°

22°   15N

Michael
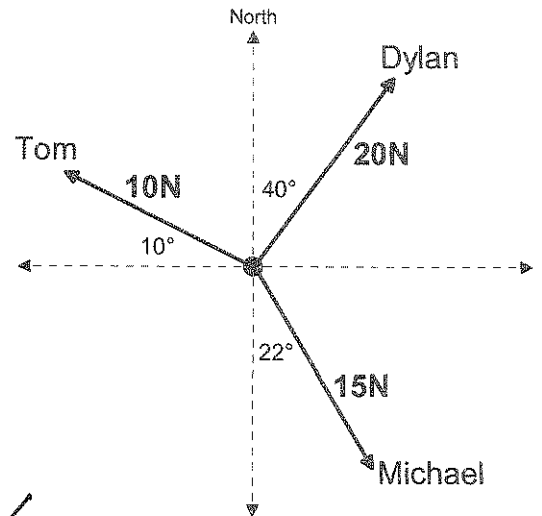
horiz

$$20\sin40° - 10\cos10° + 15\sin22°$$

$$\approx 8.627 \qquad \rightarrow \qquad ✓✓$$

Vert

$$20\cos40° + 10\sin10° - 15\cos22°$$

$$\approx 3.150 \qquad \uparrow \qquad ✓$$

$\overrightarrow{TOTAL}$   3.150

$\theta$

8.627

Magn of total force $= \sqrt{8.627^2 + 3.150^2}$

$$= 9.184 \qquad ✓$$

$$\theta = \tan^{-1}\left(\frac{3.15}{8.627}\right)$$

$$= 20.1° \qquad ✓$$

Direction   69.9°   as a   bearing ✓

(accept 70.0°)

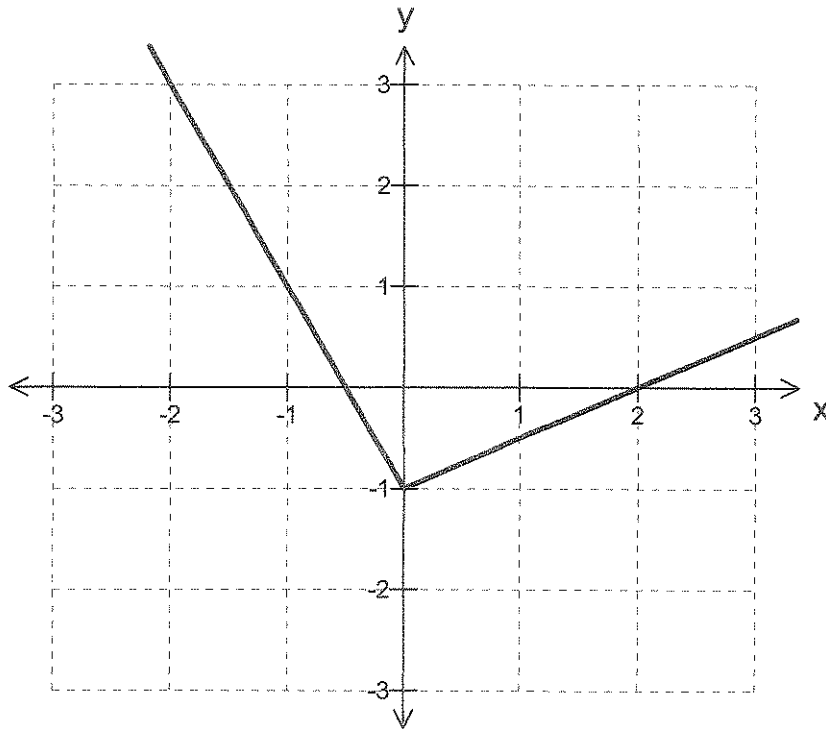with a force of   9.2 N   ✓

must round 1dp

max

−1 overall if not rounded to 1dp
(Note: This is the only question we penalise for not rounding) ✓

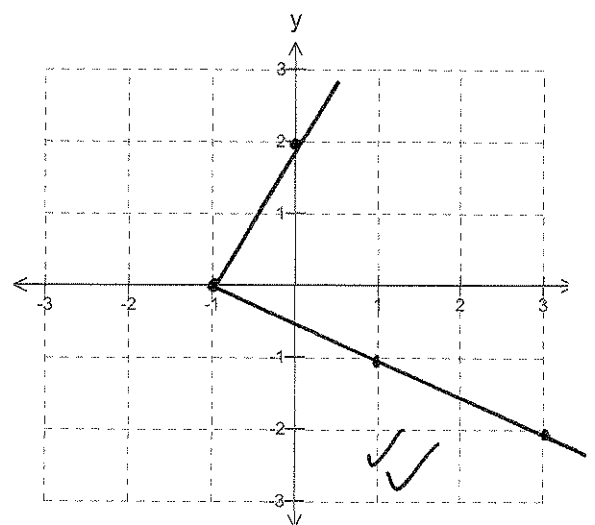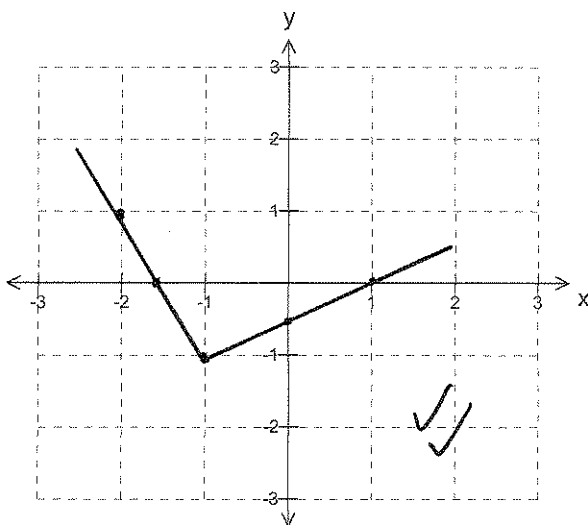## Question 9

[ 2 & 2 = 4 marks]

The graph below represents the function $y = f(x)$.



Using the axes shown below, sketch the graphs of:

a) $y = f(x+1)$

b) $y = f^{-1}(x)$



I mark for each halve

**Question 10**
[ 1 , 2 , 3 & 4 = 10 marks]

At 11am a ship is known to have the coordinates (3,-7) km and is moving with

constant velocity $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ km/hr. Its home harbour has coordinates (0,0) km.

a) Write down an expression for the ship's position vector from the harbour at time $t$ hours after 11am..

$$\begin{pmatrix} 3 \\ -7 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix} t \quad \checkmark \quad \text{or} \quad \begin{pmatrix} 3+4t \\ -7-2t \end{pmatrix}$$

b) Determine the ship's distance from the harbour at 2pm ($t=3$ hours). $(1 d_p)$

$$t=3 \quad \underset{\sim}{r} = \begin{pmatrix} 15 \\ -13 \end{pmatrix} \checkmark \quad |x| = \sqrt{15^2 + (-13)^2} \approx 19.8 \text{ Km} \checkmark$$

At 11am a tug boat leaves the harbour with a velocity $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ km/hr.

c) Determine the distance between the tug boat and the ship at 2pm. $(1 d_p)$

$$\text{tug} \quad t=3 \quad \underset{\sim}{r} = \begin{pmatrix} -9 \\ 15 \end{pmatrix} \checkmark \quad \text{ship} \quad \underset{\sim}{r} = \begin{pmatrix} 15 \\ -13 \end{pmatrix} \quad \vec{d} = \begin{pmatrix} 15 \\ -13 \end{pmatrix} - \begin{pmatrix} -9 \\ 15 \end{pmatrix} = \begin{pmatrix} 24 \\ -28 \end{pmatrix} \checkmark$$

$$d \approx 36.9 \text{ Km} \checkmark$$

d) At what time, to the nearest minute, will both the ship and the tug boat be equidistant from the harbour?

$$|x_{Tug}| = \left| \begin{pmatrix} -3t \\ 5t \end{pmatrix} \right| \qquad |r_{ship}| = \left| \begin{pmatrix} 3+4t \\ -7-2t \end{pmatrix} \right|$$

$$\sqrt{(-3t)^2 + (5t)^2} \checkmark = \sqrt{(3+4t)^2 + (-7-2t)^2} \checkmark$$

$$t \approx 4.61 \text{ hars} \checkmark$$

$$\approx 4 \text{ hars } 36.75 \text{ mins.}$$

$$\text{time} \quad 3:37 \text{ pm} \checkmark$$

$f/t$

Max $-1$ for no units
DO NOT penalise for not rounding.
(Note: This is the only question we
~~penalise for units~~

**Question 11**
[ 2, 3, 3 & 4= 12 marks]

The functions $f$, $g$ & $h$ are defined as follows.

$$f(x) = 3x + 4 \qquad g(x) = 5x^2 + 2 \qquad h(x) = \sqrt{x - 3}$$

a) Determine the natural domain and range of $f$.

Domain: $R$ ✓      Range: $R$ ✓

b) Which of the following composite functions exist over the natural domain of $f(x)$? Those that do not exist, give a reason why.

i) $g \circ f(x)$    exists

ii) $h \circ f(x)$    Does not exist

$r_f \nsubseteq d_h$    Must state range + domain for full marks

$R \quad x \geq 3$

c) For those composite functions in (b) that do exist, state the rule and the range for the natural domain of $f(x)$. (Do not simplify)

$$g \circ f(x) = 5(3x + 4)^2 + 2 \checkmark \qquad \text{no need to simplify}$$

Range:   $y \geq 2$ ✓

d) The function $t$ is defined as $t(x) = ax + b$ with $a$ & $b$ being constants.
If $g \circ t(x) = 45x^2 - 120x + 82$, determine the values of $a$ & $b$ (exact).

$$g \circ f = 5(ax + b)^2 + 2 \checkmark$$
$$= 5a^2x^2 + 10abx + 5b^2 + 2$$
$$= 45x^2 - 120x + 82$$

$$5a^2 = 45 \qquad \checkmark \qquad 5b^2 + 2 = 82$$
$$a = \pm 3 \qquad\qquad b^2 = 16$$
$$b = \pm 4$$

$$ab < 0$$

$$\therefore a = 3, \ b = -4 \qquad OR \qquad a = -3, \ b = 4$$
$$\checkmark \qquad\qquad\qquad \checkmark$$

**Question 12**
[ 2 & 4= 6marks]

a) Point A has position vector 3i – 2j. Points B and C are such that the position vector of B relative to C is 5i + 8j and the position vector of C relative to A is 9i – j. Determine the position vector of B.

$$r_B - r_C = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \qquad \checkmark \qquad r_C - r_A = \begin{pmatrix} 9 \\ -1 \end{pmatrix}$$

$$r_C = \begin{pmatrix} 9 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$$

$$r_B = \begin{pmatrix} 5 \\ 8 \end{pmatrix} + \begin{pmatrix} 12 \\ -3 \end{pmatrix} = \begin{pmatrix} 17 \\ 5 \end{pmatrix} \quad \checkmark \qquad \text{-1 for wrong vector notation}$$

b) To a person walking due North at 5 km/h the wind seems to come from the West (heading due East). To a second person walking due East at 12 km/h the wind appears to come from the South-East (heading North 45° West). Determine the true magnitude and direction of the wind. (1dp)
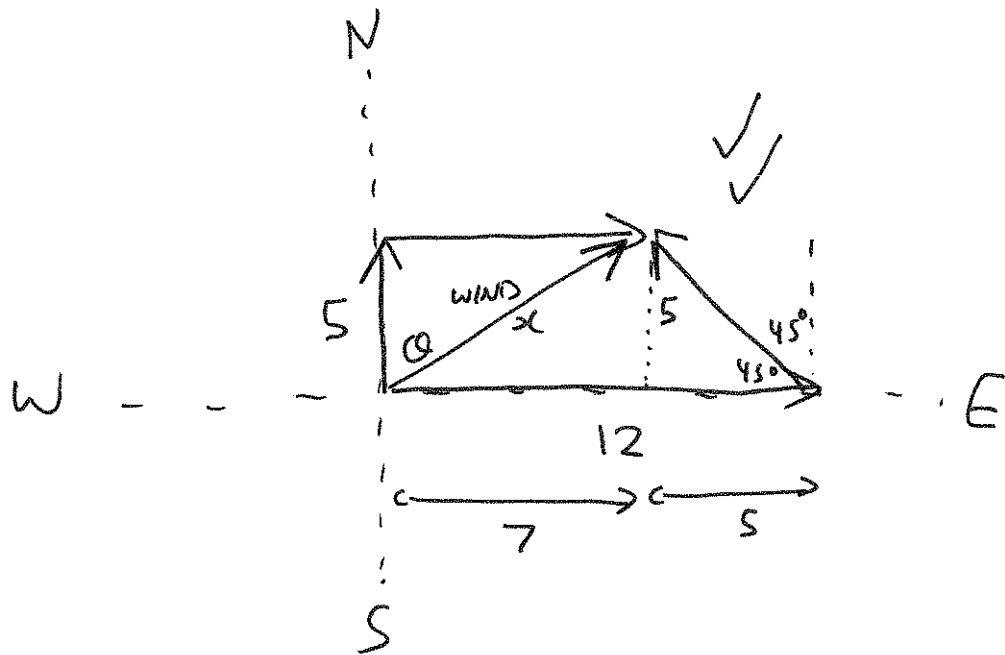


$$x = \sqrt{7^2 + 5^2} \qquad \checkmark \qquad \tan\theta = \frac{7}{5}$$
$$= 8.60 \text{ km/h} \qquad \qquad \theta = 54.5°$$

$$N \, 54.5° \, E \quad \text{at} \quad 8.6 \text{ km/h} \qquad \checkmark$$

( DO NOT penalise for not rounding nor units. )   ✓

## Question 13
[ 2, 2, 1, 2, 1 & 2 = 10 marks]

The magnitude ($M$) of earthquakes is measured on the Richter scale and is given by $M = \log_{10} A$, where $A$ is the amplitude of ground movements.

a) Restate the model in the format, $A=$.

$$A = 10^m \quad \checkmark\checkmark$$

b) Determine the amplitude of ground movements for an earthquake of magnitude (one decimal place)

    i) 6.1.

$$A = 1258925.4 \quad \checkmark$$

    ii) 4.7

$$A = 50118.7 \quad \checkmark$$

c) How many times more intense (in terms of amplitude) is a quake of magnitude 6.1 compared with one of magnitude 4.7?

$$25.1 \quad \checkmark \quad \text{or} \quad 25$$

The energy, $E$, (joules) released by an earthquake of magnitude $M$ is given by $\log_{10} E = 1.5M + 4.8$.

d) Restate the model in the format $E=$.

$$E = 10^{(1.5m + 4.8)} \quad \checkmark\checkmark$$

e) Determine the energy released by an earthquake of magnitude 6.1.

$$8.9 \times 10^{13} \text{ J} \quad \checkmark$$

f) If a nuclear bomb released $6 \times 10^{15}$ joules of energy on detonation, what would be the magnitude of an earthquake which would release the same energy?

$$\log(6 \times 10^{15}) = 1.5m + 4.8 \quad \checkmark \qquad (1dp)$$

$$M = 7.3 \quad \checkmark \qquad \text{answer only}$$
$$\text{full marks}$$

Do NOT
penalise
if not 1 dp.

**Question 14**

[ 6 marks]

A garden bed is the shape of a sector of a circle with radius 7m as shown. The perimeter of the garden bed is to be 25 metres. The top segment (shaded in the diagram) is to be filled with a special soil conditioner as native trees are to be planted in that area. The rest of the garden bed will have flowers planted in it and does not need any soil conditioner. The instructions on the bags of soil conditioner suggest a layer 5cm deep should be added to the top of the existing soil and then mixed in. Each soil conditioner bag holds 30 litres. How many bags will the gardener need to purchase? Justify your answer.
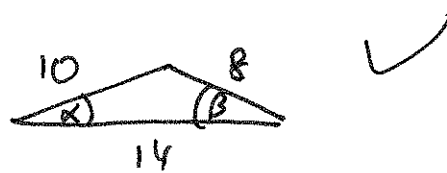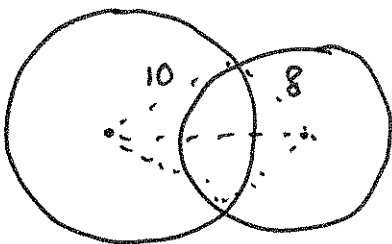
(Note: 1 kilolitre = 1 m$^3$)



Native Trees

Flowers

7 m

$$11 = r\theta$$

$$\theta = \frac{11}{7}$$
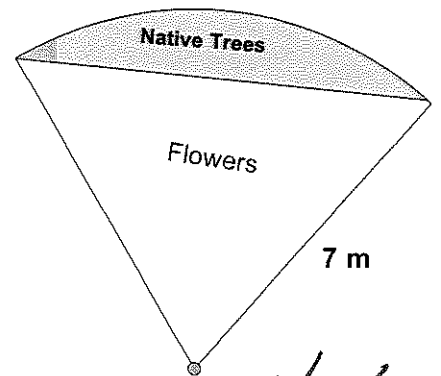
$$Area_{segment} = \frac{1}{2} 7^2 \left[ \frac{11}{7} - \sin\frac{11}{7} \right] = 14.0 \, m^2$$

$$Volume = 14 \times 0.05 \, m^3$$

$$\# \, bags = \frac{Volume \times 1000 \, litres}{30}$$

$$= 23.3$$

$$\therefore \quad 24 \, bags \, needed$$

**Question 15**

[ 6 marks]

Two circles of radii 10 cm and 8 cm have their centres 14 cm apart. Determine the perimeter of the region common to both circles, giving your answer in centimetres and correct to one decimal place.



10    8

14

$$\alpha = 34.05° \implies 0.594^R$$

$$\beta = 44.4° \implies 0.775^R$$

$$Perimeter = 10(2\alpha) + 8(2\beta)$$

$$= 10(2)(0.594) + 8(2)(0.775)$$

$$= 24.28 \, cm$$

$$= 24.3 \, cm$$

Do not penalise for not rounding nor units

**Question 16**
[ 5 & 2= 7 marks]

A plane needs to travel directly from City A (0,0) km to City B (-600,800) km. The plane travels at a speed of 75 km/hr relative to the wind. The wind's velocity is -50i + 20j km/hr. The j vector faces due North.

a) Determine the bearing of the plane's velocity relative to the wind.



$$\vec{V}_{TOTAL} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} -50 \\ 20 \end{pmatrix} = \lambda \vec{AB} = \lambda \begin{pmatrix} -600 \\ 800 \end{pmatrix}$$

$\lambda$ is a constant

$$\frac{a-50}{b+20} = \frac{-6}{8} \checkmark \quad \& \quad a^2+b^2=75^2 \checkmark$$

use CAS
$a = -19.3 \quad b = 72.4 \checkmark$
(Discard $a=64.1 \quad b=-38.9$ wrong way)

$$\tan\theta = \frac{19.3}{72.4} \quad \theta = 14.9°$$

Bearing $= 360° - 14.9°$
$= 345.1° \checkmark$
OR $345°$

max 1 out of 5
if no working

-1 if not given as bearing

b) Determine the time taken, to the nearest minute, for the plane to travel from City A to City B.

$$|\vec{AB}| = 1000 \text{ km}$$

$$|\vec{V}_{TOTAL}| = \sqrt{(a-50)^2 + (b+20)^2} = \sqrt{(-69.3)^2 + (92.4)^2}$$
$$= 115.5 \text{ km/h} \checkmark$$

f/t

$$time = \frac{1000}{115.5} \text{ hr}$$

$$= 8.67 \text{ hr}$$

$$= 8 \text{ hr } 39 \text{ mins} \checkmark \quad \text{OR} \quad (519 \text{ mins})$$
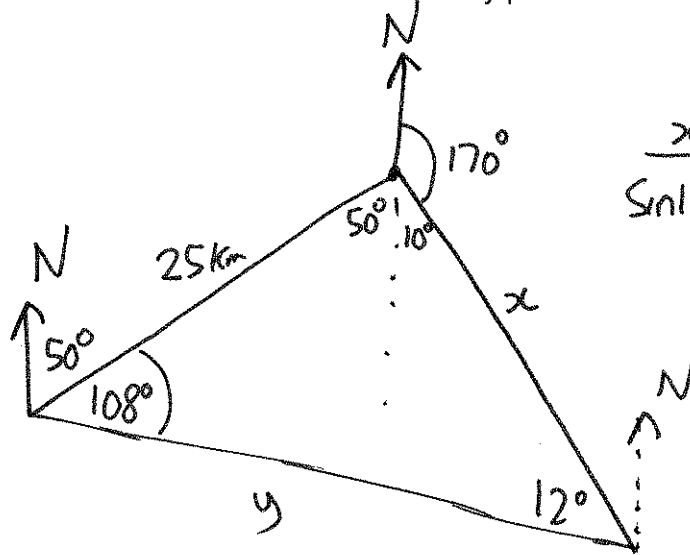
DO NOT penalise for rounding or units.

## Question 17

[ 3 & 3 = 6 marks]

Lena is working at a mine site in the middle of the desert. To make her way back to Perth, Lena must first ride a motorcycle 25 km on a bearing of 050° to the nearest airport. Lena then flies directly to Perth on a bearing of 170°. It is known that the bearing of Perth from the mine site is 158°.

a) Determine the distance Lena travels by plane.



$$\frac{x}{\sin 108°} = \frac{25}{\sin 12°}$$

$$x = 114.36 \; km$$

accept trisolve or use of CAS as working if mentioned.

b) Determine the distance between the mine site and Perth.

$$y^2 = 25^2 + x^2 - 2(25)(x)\cos 60°$$
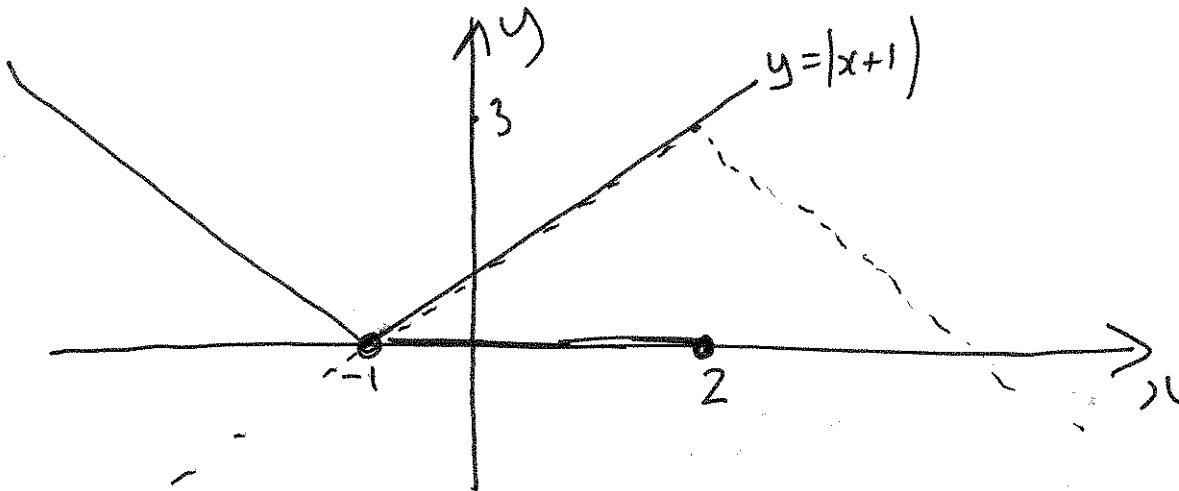
$$y = 104.1 \; km$$

Do NOT penalise for units or rounding

## Question 18
[ 6 marks]

Let $f(x) = |x + 1|$ with the domain all Real numbers.

If $f(x) = -f(x - a) + b$ is **only** true for $-1 \leq x \leq 2$ with $a$ & $b$ being constants, determine the values of $a$ & $b$.

{Hint: First consider a sketch of $f(x)$.}



✓ graph of $y = |x + 1|$

✓ drawing an upside down function

✓✓ showing overlap between -1 & 2

$a = 3$ ✓

$b = 3$ ✓

<u>NO</u> follow through, be very tough marking this!

**End of Exam !!**